Ratio adjust algos (backwards and forwards)

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1 Notations

We note by t the date of observation. Increments denote next or previous business days, so that $t-1$ is the previous business day, $t+2$ is the second business day from date t, and so on.
We are considering P, Q and R three successive futures contracts from the same series here, and we only care for now about close prices. The first contract has close price p_t on each day t, while the second one has close price q_t, and the third one has price r_t.
Finally, we consider two successive days of roll calculation t_0 (day on which we roll from P to Q) and t_1 (day on which we roll from Q to R).
As a concrete example, we take P to be SPH17 (March 2017 full-size contract on S&P 500 index), Q to be SPM17 (June 2017 contract) and R to be SPU17 (September 2017 contract). We take t_0 to be 14 March 2017 and t_1 to be 13 June 2017.

2 Forward ratio adjust

Here we are constructing a continuous sequence $\ldots, c^f_{t-2}, c^f_{t-1}, c^f_t, c^f_{t+1}, c^f_{t+2}, \ldots$ of prices from P, Q and R, rolling on day t_0.

*Cabestan Quant Research

Forward adjusting means we take

$$
\begin{array}{rlcl}
 & & \vdots & \\
 & c^f_{t_0-2} & = & p_{t_0-2} \\
 & c^f_{t_0-1} & = & p_{t_0-1} \\
\text{first roll observation} \longrightarrow & c^f_{t_0} & = & p_{t_0} \\
 & c^f_{t_0+1} & = & \frac{p_{t_0}}{q_{t_0}} q_{t_0+1} \\
 & c^f_{t_0+2} & = & \frac{p_{t_0}}{q_{t_0}} q_{t_0+2} \\
 & & \vdots & \\
 & c^f_{t_1-2} & = & \frac{p_{t_0}}{q_{t_0}} q_{t_1-2} \\
 & c^f_{t_1-1} & = & \frac{p_{t_0}}{q_{t_0}} q_{t_1-1} \\
\text{second roll observation} \longrightarrow & c^f_{t_1} & = & \frac{p_{t_0}}{q_{t_0}} q_{t_1} \\
 & c^f_{t_1+1} & = & \frac{p_{t_0}}{q_{t_0}} \frac{q_{t_1}}{r_{t_1}} r_{t_1+1} \\
 & c^f_{t_1+2} & = & \frac{p_{t_0}}{q_{t_0}} \frac{q_{t_1}}{r_{t_1}} r_{t_1+2} \\
 & & \vdots &
\end{array}
$$

If we define $\rho_0 = \frac{p_{t_0}}{q_{t_0}}$ and $\rho_1 = \frac{q_{t_1}}{r_{t_1}}$, this becomes:

$$
\begin{array}{rlcl}
 & & \vdots & \\
 & c^f_{t_0-2} & = & p_{t_0-2} \\
 & c^f_{t_0-1} & = & p_{t_0-1} \\
\text{first roll observation} \longrightarrow & c^f_{t_0} & = & p_{t_0} \\
 & c^f_{t_0+1} & = & \rho_0 q_{t_0+1} \\
 & c^f_{t_0+2} & = & \rho_0 q_{t_0+2} \\
 & & \vdots & \\
 & c^f_{t_1-2} & = & \rho_0 q_{t_1-2} \\
 & c^f_{t_1-1} & = & \rho_0 q_{t_1-1} \\
\text{second roll observation} \longrightarrow & c^f_{t_1} & = & \rho_0 q_{t_1} \\
 & c^f_{t_1+1} & = & \rho_0 \rho_1 r_{t_1+1} \\
 & c^f_{t_1+2} & = & \rho_0 \rho_1 r_{t_1+2} \\
 & & \vdots &
\end{array}
$$

Thus we multiplicatively accumulate the forward ratio adjustments ρ_i, iterating forwards in time.

The example given above is illustrated in Table 1.

3 Backward ratio adjust

Here we are constructing a continuous sequence $\ldots, c^b_{t-2}, c^b_{t-1}, c^b_t, c^b_{t+1}, c^b_{t+2}, \ldots$ of prices from P, Q and R, rolling on day t_0.

Table 1: Rolling SPH17, SPM17 and SPU17 using the forward method

	t	p_t	q_t	r_t	c_t^f	Unadj.	Cumulative Adjust
	2017-03-10	2371.8	2368.6	2365.9	2371.8	2371.8	1
	2017-03-13	2375	2371.8	2369.1	2375	2375	1
$t_0 \longrightarrow$	2017-03-14	2366.4	2363.1	2360.4	2366.4	2366.4	1
	2017-03-15	2383.7	2380.6	2377.8	2383.9	2380.6	$1.001396471 = \rho_0$
	2017-03-16		2379.1	2376.2	2382.4	2379.1	1.001396471
	2017-03-17		2375.2	2372	2378.5	2375.2	1.001396471
	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	2017-06-09		2430.5	2428.3	2433.9	2430.5	1.001396471
	2017-06-12		2428.5	2426.4	2431.9	2428.5	1.001396471
$t_1 \longrightarrow$	2017-06-13		2440	2438	2443.4	2440	1.001396471
	2017-06-14		2437.2	2435.3	2440.7	2435.3	$1.002217961 = \rho_0\rho_1$
	2017-06-15			2432.1	2437.5	2432.1	1.002217961
	2017-06-16			2431	2436.4	2431	1.002217961

Backward adjusting means we take

$$
\begin{array}{rlcl}
 & & \vdots & \\
 & c^b_{t_0-2} & = & \frac{r_{t_1}}{q_{t_1}}\frac{q_{t_0}}{p_{t_0}}p_{t_0-2} \\
 & c^b_{t_0-1} & = & \frac{r_{t_1}}{q_{t_1}}\frac{q_{t_0}}{p_{t_0}}p_{t_0-1} \\
\text{first roll observation} \longrightarrow & c^b_{t_0} & = & \frac{r_{t_1}}{q_{t_1}}q_{t_0} \\
 & c^b_{t_0+1} & = & \frac{r_{t_1}}{q_{t_1}}q_{t_0+1} \\
 & c^b_{t_0+2} & = & \frac{r_{t_1}}{q_{t_1}}q_{t_0+2} \\
 & & \vdots & \\
 & c^b_{t_1-2} & = & \frac{r_{t_1}}{q_{t_1}}q_{t_1-2} \\
 & c^b_{t_1-1} & = & \frac{r_{t_1}}{q_{t_1}}q_{t_1-1} \\
\text{second roll observation} \longrightarrow & c^b_{t_1} & = & r_{t_1} \\
 & c^b_{t_1+1} & = & r_{t_1+1} \\
 & c^b_{t_1+2} & = & r_{t_1+2} \\
 & & \vdots &
\end{array}
$$

Table 2: Rolling SPH17, SPM17 and SPU17 using the backward method

	t	p_t	q_t	r_t	c_t^b	Unadj.	Cumulative Adjust
	2017-03-10	2371.8	2368.6	2365.9	2366.6	2371.8	$0.99778695 = \sigma_0\sigma_1$
	2017-03-13	2375	2371.8	2369.1	2369.7	2375	0.99778695
$t_0 \longrightarrow$	2017-03-14	2366.4	2363.1	2360.4	2361.2	2366.4	0.99778695
	2017-03-15	2383.7	2380.6	2377.8	2378.6	2380.6	$0.99918033 = \sigma_1$
	2017-03-16		2379.1	2376.2	2377.1	2379.1	0.99918033
	2017-03-17		2375.2	2372	2373.3	2375.2	0.99918033
	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	2017-06-09		2430.5	2428.3	2428.5	2430.5	0.99918033
	2017-06-12		2428.5	2426.4	2426.5	2428.5	0.99918033
$t_1 \longrightarrow$	2017-06-13		2440	2438	2438	2440	0.99918033
	2017-06-14		2437.2	2435.3	2435.3	2435.3	1
	2017-06-15			2432.1	2432.1	2432.1	1

If we define $\sigma_0 = \frac{q_{t_0}}{p_{t_0}}$ and $\sigma_1 = \frac{r_{t_1}}{q_{t_1}}$, this becomes:

$$
\begin{array}{rlcl}
 & & \vdots & \\
 & c^b_{t_0-2} & = & \sigma_0\sigma_1 p_{t_0-2} \\
 & c^b_{t_0-1} & = & \sigma_0\sigma_1 p_{t_0-1} \\
\text{first roll observation} \longrightarrow & c^b_{t_0} & = & \sigma_1 q_{t_0} \\
 & c^b_{t_0+1} & = & \sigma_1 q_{t_0+1} \\
 & c^b_{t_0+2} & = & \sigma_1 q_{t_0+2} \\
 & & \vdots & \\
 & c^b_{t_1-2} & = & \sigma_1 q_{t_1-2} \\
 & c^b_{t_1-1} & = & \sigma_1 q_{t_1-1} \\
\text{second roll observation} \longrightarrow & c^b_{t_1} & = & r_{t_1} \\
 & c^b_{t_1+1} & = & r_{t_1+1} \\
 & c^b_{t_1+2} & = & r_{t_1+2} \\
 & & \vdots &
\end{array}
$$

Thus we multiplicatively accumulate the backward ratio adjustments ρ_i, iterating backwards in time.

The example given above is illustrated in Table 2.